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COMMENT

Comment on 'Correlative amplitude-operational phase entanglement embodied by the EPR-pair eigenstate $|\eta\rangle$'

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Abstract

In a recent paper in this journal Fan (Fan H 2002 *J. Phys. A: Math. Gen.* **35** 1007) discards the possibility of using a genuine phase-difference operator to investigate number-phase entanglement because of the lack of unitarity of the Susskind–Glogower phase operators. However, Fan overlooked the existence of a bona fide unitary operator exponential of the phase difference. Here we find the amplitude–phase maximally entangled states as the simultaneous eigenstates of the total number and the phase-difference operators.

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In a recent work Fan investigated the existence of number-phase entangled states [1]. After examining the quantum description of these variables the author abandons the possibility of using the phase-difference variable because the operator exponential of the phase difference that the author employs (the product of Susskind–Glogower phase operators [2]) is not unitary. Instead, the author considers another operator that no longer represents the genuine phase-difference variable, but a noisy version of a single mode phase [3].

However, it should be stressed that it is possible to represent the phase difference by a Hermitian operator and the exponential of the phase difference by a unitary operator. This was demonstrated in [4, 5], rediscovered in [6] and verified experimentally in [7]. Its properties have been further examined in [8]. This operator, which was overlooked in [1], should be crucial for investigating number-phase entanglement. In this comment, we complete the approach of Fan showing that this operator serves to construct genuine maximally entangled states with respect to the amplitude and phase variables.

The unitary operator exponential of the phase difference $\mathcal{E}$ is naturally defined by the unitary solution of the two-mode polar decomposition

$$a_1 a_2^\dagger = \sqrt{a_2^\dagger a_2 a_1 a_1^\dagger}\,\mathcal{E} = \mathcal{E}\sqrt{a_1^\dagger a_1 a_2 a_2^\dagger} \tag{1}$$

where a_1, a_2 are the annihilation operators for the corresponding modes. It has been shown that this equation admits unitary solutions $\mathcal{E}^\dagger\mathcal{E} = \mathcal{E}\mathcal{E}^\dagger = 1$ [3, 5]. It should be noted that the product of Susskind–Glogower operators used in [1] is a non-unitary solution of the same polar decomposition (1). On the other hand, the operational phase operator finally considered in [1] satisfies a different polar decomposition that is not proportional to $a_1 a_2^\dagger$, so it represents a phase-angle variable different from what is usually understood as the phase difference.

For continuous Cartesian conjugate variables X_j, P_j, with $j = 1, 2$ and $[X_j, P_{j'}] = \mathrm{i}\delta_{j,j'}$, $[X_j, X_{j'}] = [P_j, P_{j'}] = 0$, the common eigenstates of the commuting operators $X_1 - X_2$ and $P_1 + P_2$ are maximally entangled states for the X, P variables [9]. In our case the conjugate variables are number and phase. Accordingly, it can be expected that the simultaneous eigenstates of the phase difference $\mathcal{E}$ and the number sum $a_1^\dagger a_1 + a_2^\dagger a_2$ (total number) are maximally entangled states.

It can be seen that their common eigenvectors are the states $|N, \phi_{N,r}\rangle$

$$\mathcal{E}|N, \phi_{N,r}\rangle = \mathrm{e}^{\mathrm{i}\phi_{N,r}}|N, \phi_{N,r}\rangle \qquad \left(a_1^\dagger a_1 + a_2^\dagger a_2\right)|N, \phi_{N,r}\rangle = N|N, \phi_{N,r}\rangle \tag{2}$$

with

$$|N, \phi_{N,r}\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^{N} \mathrm{e}^{\mathrm{i}n\phi_{N,r}}|n\rangle_1|N-n\rangle_2 \tag{3}$$

where $|n\rangle_j$ are number states in the corresponding mode,

$$\phi_{N,r} = \phi_{N,0} + \frac{2\pi}{N+1}r \qquad r = 0, 1, \ldots, N \tag{4}$$

and $\phi_{N,0}$ are arbitrary phases.

It can be deduced by simple inspection that $|N, \phi_{N,r}\rangle$ are maximally entangled states, as conjectured above. This was already noted in [10] and applied to quantum teleportation in [11]. This completes the key of this comment: to demonstrate the existence of genuine amplitude–phase entanglement derived from a bona fide phase-difference operator overlooked in [1].

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